Algebraic Operations (Multi-image Point Operations)

Week-11

https://www.broadridge.com/intl/about/optimize-operations
Introduction

• For multi-spectral or, more generally, multi-layer images, algebraic operations such as **the four basic arithmetic operations** (+, -, x, ÷), logarithmic, exponential, sin and tan can be applied to the digital numbers (DNs) of different bands for each pixel to produce a new image.

• Such processing is called **image algebraic operation**.

• Algebraic operations are performed pixel by pixel among DNs of spectral bands (or layers) for each pixel **without involving** neighbourhood pixels.

• They can therefore be considered as **multi-image point operations** defined as below:

\[ y = f(x_1, x_2, \ldots, x_n) \]

\[ n = \text{number of bands} \]
Introduction

• To start with, let us consider the four basic arithmetic operations: addition, subtraction, multiplication and division.

• In multi-image point operations, arithmetic processing is sometimes the same as matrix operations, such as addition and subtraction, but sometimes totally different from and much simpler than matrix operations, such as image multiplication and division.

• As the image algebraic operation is position relevant, that is, pixel-to-pixel based, we can generalise the description.
Image addition

• This operation produces a **weighted summation** of two or more images:

\[ Y = \frac{1}{k} \sum_{i=1}^{n} w_i X_i \quad w_i = \text{weight of image } X_i \]

\[ k = \text{scaling factor} \]

• An important application of image addition is **to reduce noise** and increase the **signal-to-noise ratio (SNR)**.

• Suppose each image band of an n band **multispectral image** is contaminated by an additive noise source \( N_i(i=1,2,...,n) \).

• The noise pixels are not likely to occur **at the same position** in different bands, and thus a noise pixel DN in band \( i \) will be averaged with the non-noise DNs in the other \( n-1 \) bands.

• As a result the noise will be **largely suppressed**.
Example 1:
Image addition

• It is proved from signal processing theory that of n duplications of an image, each contaminated by the same level of random noise, the SNR of the sum image of these n duplications equals the square root n times the SNR of any individual duplication:

\[ SNR_y = \sqrt{n} \cdot SNR_i \]

• This equation implies that for an n bands multi-spectral image, the summation of all the bands can increase SNR by about \( n^{1/2} \) times.

• For instance, if we average bands 1–4 of a Landsat TM image, the SNR of this average image is about two times (\( 4^{1/2} = 2 \)) of that of each individual band.
Example 2:
Image subtraction (differencing)

- **Image subtraction** produces a difference image from two input images:

\[ Y = \frac{1}{k} \left( w_i X_i - w_j X_j \right) \]

- The weights \( w_i \) and \( w_j \) are important to ensure that balanced differencing is performed.
- If the brightness of \( X_i \) is significantly higher than that of \( X_j \), for instance, the difference image \( X_i - X_j \) will be dominated by \( X_i \), and as a result, the true difference between the two images will not be effectively revealed.
- To produce a ‘fair’ difference image, **balance contrast enhancement technique** (BCET) or histogram matching (matching the histogram of \( X_i \) to that of \( X_j \)) may be applied as a pre-processing step.
- Whichever method is chosen, the differencing that follows should then be performed with equal weighting (\( w_i = w_j = 1 \)).
Example 3:
Subtraction is one of the simplest and most effective techniques for **selective spectral enhancement**, and it is also useful for **change detection** and **removal of background illumination bias**.

However, in general, subtraction reduces the image information and decreases image **SNR** because it removes the common features while retaining the **random noise** in both images.
Image subtraction (differencing)

- Band differences of multi-spectral images are successfully used for studies of vegetation, land use and geology.
- As shown in next Fig., TM band difference of TM3-TM1 (R-B) highlights iron oxides; TM4-TM3 (NIR-Red) enhances vegetation; and TM5-TM7 is effective for detecting hydrated (clay) minerals.
- These three difference images can be combined to form an **RGB colour composite image** to highlight iron oxides, vegetation and clay minerals in red, green and blue, as well as other ground objects in various colours.
- In many cases, subtraction can achieve similar results to division (ratio) and the operation is simpler and faster.
(a) TM3-TM1 highlights red features often associated with iron oxides;
(b) TM4-TM3 detects the diagnostic ‘red edge’ features of vegetation;
(c) TM5-TM7 enhances the clay and hydrate mineral absorption features in the short-wave infrared (SWIR) spectral range;
(d) the colour composite of TM3-TM1 in red, TM4-TM3 in green and TM5-TM7 in blue that highlights iron oxide, vegetation and clay minerals in red, green and blue colours.
Image subtraction (differencing)

• The image subtraction technique is also widely used for **background noise removal** in microscopic image analysis.

• An image of the background illumination field (as a reference) is captured before the target object is placed in the field.

• The second image is then taken with the target object in the field.

• The **difference image between the two** will retain the target while the effects of the illumination bias and background noise are cancelled out.
Image multiplication

- **Image multiplication** is defined as below:

\[ Y = X_i \cdot X_j \]

- Here the image multiplication is performed pixel by pixel; at each image pixel, its **band** \( i \) DN is multiplied with **band** \( j \) DN.
- This is fundamentally **different from** matrix multiplication.
- A digital image is a two-dimensional (2D) array, but it is not a matrix.
- A product image of multiplication often has much greater DN range than the dynamic range of the display devices and thus needs to be **re-scaled** before display.
- Most image processing software packages can display any image based on its actual value range, which is then fit into a 0–255 display range.
Example 4:
Image multiplication

• One application of multiplication is **masking**.
• For instance, if \( X_i \) is a mask image composed of DN values 0 and 1, the pixels in image \( X_j \) corresponding to 0 in \( X_i \) will become 0 (masked off) and others will remain unchanged in the product image \( Y \).
• This operation could be achieved more efficiently using a logical operation of a given condition.
• Another application is **image modulation**.

![Image multiplication](image)
Image modulation

- For instance, **topographic features** can be added back to a colour coded classification image by using a panchromatic image (as an intensity component) to modulate the three colour components (red, green and blue) of the classification image as below:

1. Produce red (R), green (G) and blue (B) component images from the colour coded classification image.

2. Use the relevant panchromatic image (I) to modulate R, G and B components: $R \times I$, $G \times I$ and $B \times I$.

3 Colour composition using $R \times I$, $G \times I$ and $B \times I$.

- This process is, in some image processing software packages, automated by draping a colour coded classification image on an intensity image layer.
Multiplication for image modulation:
(a) a colour coded classification image; and
(b) the intensity modulated classification image.
Image division (ratio)

- **Image division** is a very popular technique, also known as an **image ratio**.

- The operation is defined as:

\[ Y = \frac{X_i}{X_j} \]

- In order to carry out image division, certain protection is needed to **avoid overflow**, in case a number is divided by zero.

- A commonly used trick in this context is to **change 0 to 1** whenever a 0 becomes a divisor.

- A better approach is to shift the value range of the **denominator image upwards, by 1**, to avoid zero.

- For an 8-bit image, this shift changes the image DN range from 0–255 to 1–256 that just exceeds 8 bits.

- This was a problem for the older generation of image processing systems before the 1990s but is no longer a problem for most modern image processing software packages where the image processing is performed based on the double precision floating point data type.
Example 5:
Image division (ratio)

• A ratio image **Y is an image of real numbers instead of integers.**

• If both \(X_i\) and \(X_j\) are 8-bit images, the possible **maximum value range** of \(Y\) is \([1/255, 255]\).

• Instead of a much simpler notation, \([0, 255]\), we deliberately write the value range in such a way to emphasise that the value range \([1/255, 1]\) may contain just as much information as that in the much wider value range \((1, 255]\).

• A popular approach for displaying a ratio image on an 8-bit/pixel/channel display system is to scale the image into a 0–255 DN range, and many image processing software packages may perform the **operation automatically**.

• This may result in up to 50% information loss because the information recorded in value range \([1/255, 1]\) could just be in a **few DN levels**.
Example 6:

• One of the most important uses of division is in change detection:
Image division (ratio)

- If we consider an image ratio as a coordinate transformation from a Cartesian coordinate system to a polar coordinate system (Fig. 3.3) rather than a division operation, then:

\[
Y = \frac{X_i}{X_j} = \tan(\alpha) \quad \alpha = \arctan\left(\frac{X_i}{X_j}\right)
\]

- Ratio image Y is actually a tangent image of the angle \(\alpha\).
- The information of a ratio image is evenly presented by angle \(\alpha\) in value range \([0,\pi/2]\) instead of by \(Y \tan(\alpha)\) in value range \([0,255]\).
- Therefore, to achieve a ‘fair’ linear scale stretch of a ratio image, it is necessary to convert Y to \(\alpha\) by this eqn.
- A linear scale can then be performed as

\[
\beta = 255 \frac{\alpha - \text{Min}(\alpha)}{\text{Max}(\alpha) - \text{Min}(\alpha)}
\]
Example 7:
After all, the above transform may not be always necessary.

Ratios are usually designed to highlight the target features as high ratio DNs.

In this case, the direct stretch of ratio image Y may enhance the target features well but at the expense of the information represented by low ratio DNs.

From this sense and as an example, it is important to notice that although ratios TM1/TM3 and TM3/TM1 are reciprocals of one another mathematically and so contain the same information, they are different in terms of digital image display after linear scale!

Remember, when you design a ratio, make sure that the target information is highlighted by high values in the ratio image.
• **Ratio** is an effective technique to selectively enhance spectral features.

• Ratio images derived from different band pairs are often used to generate ratio colour composites in an RGB display.

• For instance, a colour composite of **TM5/TM7** (blue), **TM4/TM3** (green) and **TM3/TM1** (red) may highlight clay mineral in blue, vegetation in green and iron oxide in red.

• It is interesting to compare Figures to notice the similarity between differencing and ratio techniques for selective enhancement.

• Many indices, such as the **Normalised Difference Vegetation Index (NDVI)**, have been developed based on both differencing and ratio operations.
Ratio images and ratio colour composite: (a) the ratio image of TM3/TM1; (b) the ratio image of TM4/TM3; (c) the ratio image of TM5/TM7; and (d) the ratio colour composite of TM5/TM7 in blue, TM4/TM3 in green and TM3/TM1 in red.
Index derivation and supervised enhancement

• **Infinite combinations** of algebraic operations can be derived from basic arithmetic operations and algebraic functions.

• **Aimless combinations** of algebraic operations may mean an endless and potentially fruitless game, that is, you may spend a very long time without achieving any satisfactory result.

• Alternatively, you may happen upon a visually impressive image without being able to explain or interpret it.

• To design a meaningful and effectively combined operation, the knowledge of the **spectral properties of targets** is essential.
Index derivation and supervised enhancement

• The formulae should be composed on the basis of spectral or physical principles and designed for the enhancement of particular targets; these are then referred to as spectral indices, such as the NDVI (Normalized Difference Vegetation Index).

• An index can be considered as supervised enhancement.

• Here we briefly introduce a few commonly used examples of indices based on Landsat TM/ETM+ image data.

• You may design your own indices for a given image processing task based on spectral analysis.

Image spectral signatures of vegetation, red soil and clay minerals
Vegetation indices

- As shown in Fig., healthy vegetation has a high reflection peak in the **near infrared (NIR)** and an absorption trough **in the red**.

- If we could see **NIR**, vegetation would be NIR rather than green.

- This significant difference between **red** and **NIR** bands is known as the red edge; it is a unique spectral property that makes vegetation different from all other ground objects.

- Obviously, this diagnostic spectral feature of vegetation can be very effectively enhanced by differencing and ratio operations.

- Nearly all the vegetation indices are designed to highlight the **red edge** in one way or another.
Vegetation indices

• The NDVI is one of the most popular vegetation indices:

\[
NVDI = \frac{NIR - Red}{NIR + Red}
\]

• This index is essentially a difference between NIR and Red spectral band images.
• The summation of NIR and Red in the denominator is a factor to normalize the NDVI to a value range \([-1,1]\) by ratio.
• The NDVI for TM imagery:

\[
Y = \frac{TM_4 - TM_3}{TM_4 + TM_3}
\]

• Vegetation can also be enhanced using a ratio index:

\[
Y = \frac{NIR - \text{Min}(NIR)}{\text{Red} - \text{Min}(\text{Red}) + 1}
\]

\[
Y = \frac{TM_4 - \text{Min}(TM_4)}{TM_3 - \text{Min}(TM_3) + 1}
\]
Vegetation indices

• The effect of the subtraction of the band minimum is to roughly remove the added constants of atmospheric scattering effects so as to improve topography suppression by ratio.

• The value of 1 added to the denominator is to avoid a zero value.

• Figure illustrates these vegetation indices derived from a TM image.
Vegetation indices

(a) Landsat TM NDVI; and (b) vegetation ratio images.
Iron oxide ratio index

- **Iron oxides and hydroxides** are some of the most commonly occurring minerals in the natural environment.
- They appear red or reddish brown to the naked eye because of high reflectance in red and absorption in blue (Fig. 3.6).
- Typical red features on land surfaces, such as red soils, are closely associated with the presence of iron bearing minerals.
- We can enhance iron oxides using the ratio between red and blue spectral band images:

\[
Y = \frac{\text{Red} - \min(\text{Red})}{\text{Blue} - \min(\text{Blue}) + 1}
\]

\[
Y = \frac{\text{TM} 3 - \min(\text{TM} 3)}{\text{TM} 1 - \min(\text{TM} 1 + 1)}
\]
Iron oxide ratio index

TM iron oxide ratio index image
TM clay (hydrated) mineral ratio index

- **Clay minerals** are characteristic of hydrothermal alteration in rocks and are therefore very useful indicators for mineral exploration using remote sensing.

- The diagnostic spectral feature of clay minerals, which differentiates them from unaltered rocks, is that they all have strong absorption in the spectral range around 2.2 μm (corresponding to TM band 7) in contrast to high reflectance in the spectral range around 1.65 μm (corresponding to TM band 5), as shown in Fig. 3.6.

- Thus clay minerals can be generally enhanced by the ratio between these two SWIR bands:

\[
Y = \frac{TM\ 5 - Min(TM\ 5)}{TM\ 7 - Min(TM\ 7) + 1}
\]
TM clay (hydrated) mineral ratio index

TM clay mineral ratio index image
Standardization and logarithmic residual

• A typical example of a combined algebraic operation is the so-called standardisation:

$$Y_i = \frac{X_i}{\frac{1}{k} \sum_{\lambda=1}^{k} X_\lambda}$$

where $X_i$ represents band $i$ image, $Y_i$ the standardized band $i$ image and $k$ the total number of spectral bands.

• This ratio type operation can suppress topographic shadows.

• The denominator in the formula is the average image of all the bands of a multi-spectral image; this allows the ratio for every band to be produced using the same divisor.

• The standardization enables the spectral variation among different bands, at each pixel, to be better enhanced using the ratio to the same denominator.